C. A. ANSELMI - L. FINO

A MATHEMATICAL MODEL FOR THE STRESS ANALYSIS OF MASONRY

1. *Introduction*

An approach to develop the stress analysis of a masonry, considered as a " no tension material ", discretized by finite elements, is here retaken.

In the approximation, we limit ourselves to " linear theory " and to bidimensional elastic continuum.

The problem which we intend to consider is a " unilateral problem " whose formulation leads to inequalities: it is well known that the solution can be obtained studying a non linear programming problem.

Here a non standard approach, founded on the force method and on a reticular model, is proposed; this model has been obtained considering that:

— for " no tension material " it is not possible to use equilibrium models by variable stress states, because it would be necessary to impose the condition on the sign of principal stresses in every point of the generical element;

— the adoption, of generical equilibrium models, with constant stress states sign would impose the introduction of non linear inequalities.

On the contrary, by the reticular model, we have to impose a linear condition for the sign alone on the stresses in the bars. The solution is obtained by the stationarity requirements for the complementary energy with the respect of the equilibrium conditions and the inequalities assigned.

In this manner, the problem becomes a simple linear programming problem.

2. *The model*

Every continuum, having a generical geometry, can be assumed as a reticular model. In fact, the continuum can be divided into strips (prisms) by fictitious lines; the strips are assumed to be connected to one anothers along a discrete number of nodal lines which coincide with the short boundaries of the strip.

For semplicity, we limit ourselves to a rectangular panel with unit thickness, having generical constraints and subjected on the edges to external surface loads, and in every points to body forces.

In the reticular model we will put concentrated loads, equivalent to external surface loads, on the nodal points, and the same it will be done for the body forces.

We divide the panel in $(2n + 1)$ horizontal strips and in $(2M + 1)$ vertical strips, having width s_2 and s_1 respectively (Fig. 1). The pins of the reticular model are assumed coincident with the centroids of the rectangular elements of the strips having odd index; the bars are assumed or to have width s_1 and short edges coincident with diagonals of the rectangular elements containing the pins, or, if they are diagonal bars, to have width $2s_1 s_2 \sqrt{s_1^2+s_2^2}$ and short edges as before Fig. 2 a,b).

Obviously, in this manner the strips have some common triangular portions.

3. *Equilibrium conditions*

With this model we have to write alone equilibrium conditions on the pins. If we consider the generical point i (Fig. 3 a, b), the axial loads in the bars are:

$$(3.1)\qquad \begin{array}{ll} N_{i,i-1} = \sigma_{i,i-1}\, s_2 & N_{i,i+1} = \sigma_{i,i+1}\, s_2 \\ N_{i,i+n-1} = \sigma_{i,i+n-1}\, \dfrac{2s_1s_2}{\sqrt{s_1^2+s_2^2}} & N_{i,i-n+1} = \sigma_{i,i-n+1}\, \dfrac{2s_1s_2}{\sqrt{s_1^2+s_2^2}} \\ N_{i,i+n} = \sigma_{i,i+n}\, s_1 & N_{i,i-n} = \sigma_{i,i-n}\cdot s_1 \end{array}$$

Therefore the equilibrium conditions are:

$$(3.2)\qquad \left\{ \begin{array}{l} N_{i,i+1} - N_{i,i-1} + \left(N_{i,i-n+1} - N_{i,i+n-1}\right)\dfrac{s_1}{\sqrt{s_1^2+s_2^2}} = F_{i1} \\ N_{i,i-n} - N_{i,i+n} + \left(N_{i,i-n+1} - N_{i,i+n-1}\right)\dfrac{s_2}{\sqrt{s_1^2+s_2^2}} = F_{i2} \end{array} \right.$$

where F_i are body forces or external forces.

Using the (3.1), the (3.2) are written as function of the stresses:

$$(3.3)\qquad \left\{ \begin{array}{l} \left(\sigma_{i,i+1} - \sigma_{i,i-1}\right) + \dfrac{2s_1^2}{s_1^2+s_2^2}\left(\sigma_{i,i-n+1} - \sigma_{i,i+n-1}\right) = f_{i1} \\ \left(\sigma_{i,i-n} - \sigma_{i,i+n}\right) + \dfrac{2s_2^2}{s_1^2+s_2^2}\left(\sigma_{i,i-n+1} - \sigma_{i,i+n-1}\right) = f_{i2} \end{array} \right.$$

where it is:

$$f_{i1} = F_{i1}/s_2 \quad , \quad f_{i2} = F_{i2}/s_1$$

Finally, the system of equilibrium conditions is:

$$(3.4)\qquad \underset{\sim}{A}\,\underset{\sim}{\sigma} = \underset{\sim}{f}$$

where σ and f are the unknown and the constant vectors respectively.

As for the reticular model generally we have a system with fewer equations than unknowns, we can put:

$$\underset{\sim}{\sigma}^T = \left[\, \sigma_o^T \mid x_a^T \mid x_s^T \,\right]$$

where σ_o is the unknown vector for the bars of the isostatic model, x_a the unknown vector for the redundant bars, and x_s the reaction vector for the redundant constraints.

Consequently, the (3.4) becomes:

$$(3.4')\qquad \underset{\sim}{A}_1\,\underset{\sim}{\sigma}_o + \underset{\sim}{A}_2\,\underset{\sim}{x}_a + \underset{\sim}{A}_3\,\underset{\sim}{x}_s = \underset{\sim}{f}$$

and we can write:

$$(3.5)\qquad \underset{\sim}{\sigma}_o = \underset{\sim}{A}_1^{-1}\left(\underset{\sim}{f} - \underset{\sim}{A}_2\,\underset{\sim}{x}_a - \underset{\sim}{A}_3\,\underset{\sim}{x}_s\right)$$

4. *Complementary energy*

We can use the theorem of minimum complementary energy to look for solutions able to satisfy the compatibility conditions together the equations of equilibrium.

Considering the direct energy for every bar and the mutual energy for the common portions of the bars, we can write the complementary energy for the whole structure:

$$(4.1)\qquad E_c = \frac{1}{2}\left[\underset{\sim}{\sigma}_o^T \underset{\sim}{B}_{11} \underset{\sim}{\sigma}_o + \underset{\sim}{x}_a^T \underset{\sim}{B}_{22} \underset{\sim}{x}_a + \underset{\sim}{\sigma}_o^T \underset{\sim}{B}_{12} \underset{\sim}{x}_a + \underset{\sim}{x}_a^T \underset{\sim}{B}_{21} \underset{\sim}{\sigma}_o\right] - \underset{\sim}{x}_s^T \underset{\sim}{C}$$

In the expression between brackets the first two terms concern the isostatic scheme and the redundant bars respectively, and the others concern the mutual energy, while the last ter min (4.1) concernens the given displacements on the boundary.

Using the (3.5) finally we have:

$$E_c = \frac{1}{2}\left[f^T K_{ff} f + x_a^T K_{aa} x_a + x_s^T K_{ss} x_s + f^T K_{fa} x_a + x_a^T K_{af} f + f^T K_{fs} x_s + x_s^T K_{sf} f + x_a^T K_{as} x_s + x_s^T K_{sa} x_a \right] - x_s^T C \tag{4.1'}$$

As it is necessary to satisfy also the conditions:

$$\sigma_0 = A_1^{-1}(f - A_2 x_a - A_3 x_s) \leq 0 , \qquad x_a \leq 0 \tag{4.2}$$

the problem we have to solve is a problem of constrained extrema. Precisely, it needs to solve a quadratic programming problem or, by the Kuhn and Tucker theorem, a linear programming problem.

5. *Final remarks*

The Kuhn and Tucker theorem leads to the following conditions:

$$\sigma^* = K\lambda + b \geq 0 \; ; \quad \lambda \geq 0 , \quad \lambda^T \sigma^* = 0 \tag{5.1}$$

where

$$\sigma^{*T} = [-\sigma_0^T \,\vdots\, -x_a^T]$$

and K is a symmetric matrix, λ has to be considered a displacement vector associated to stresses (fractures in the bars), and the third condition implies that where a fracture appears in the material there the tension has to be zero or viceversa.

This problem, known as a " linear complementarity problem " admits a unique solution if and only if K is a positive definite matrix; the solution may be not unique if K is a positive semidefinite matrix. Finally, the problem will have solution if and only if the system of the conditions (3.4') and (4.2) has solutions.

The method bears some similarity with " relaxation approaches " because the making zero the tensile stresses obtained in some bars, after the elastic analysis, means to admit a fracture in the bars, to apply some

" restraining " forces, and to reanalyse the continuum for the effect of such forces, until all tensile stresses will be reduced to a negligible quantity.

In analytic terms, all this procedure extricates itself automatically using " simplex method " for the solution of the problem (5.1).

6. *Numerical application*

The panel in fig. 4, having a thickness of 0,6 m, elastic constants $E = 10^6$ tn/mq and $\nu = 0{,}1$, and mass density $\chi = 1{,}6$ t/m³ is here studied.

In fig. (5 a, b) a coarse discretization of the continuum is shown together the conditions on the constraints and the forces. In fig. 6 it is possible to note, by the stresses determined, the classic development of the isostatic lines.

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THEME: STRUCTURES

TITRE: UN MODELE MATHEMATIQUE POUR L'ANALYSE DES EFFORTS EN MACONNERIE

RESUME:

Nous nous trouvons en présence d'une nouvelle tentative en vue de développer l'analyse des efforts dans le cas de la maçonnerie, matériau considéré peu capable d'absorber des efforts de tension, en éléments isolés.

Le modèle appliqué est « réticulaire » ce qui permet d'obtenir une solution grâce à la condition stationnaire de l'énergie complémentaire associée à un système d'inégalités.

Le problème que nous devons résoudre est classique. Il s'agit de la programmation quadratique qui devient un problème de complémentarité linéaire si l'on applique le théorème de Kuhn et Tucker. Il peut donc être résolu par la « méthode simple ».

Cette dernière méthode, en termes physiques, permet d'appliquer certaines forces « restrictives » dans les barres du modèle sujettes à la tension et de réanalyser la continuité produite par ces forces jusqu'à ce que tous les efforts de tension se réduisent à une quantité négligeable.

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SUBJECT: STRUCTURES

TITLE: A MATHEMATICAL MODEL FOR THE STRESS ANALYSIS OF MASONRY

SUMMARY:

An approach to stress analysis of masonry, considered as a " no tension material " is presented in this communication.

The model used is a reticular model which permits the solution of problems by studying the state of equilibrium of complementary energy in conjunction with a mathematical system of inequalities.

The problem we have to solve is a classical problem of quadratic programming which becomes one of linear complementarity according to the Kuln and Trucker theorem: it may therefore be solved by the " simple method ".

In terms of physics, this last method allows to apply restraining forces in the model bars which are subjected to tensile stresses and to reanalyse the continuum to see the effect of these forces until all tensile stresses are reduced to a negligible quantity.

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TEMA: ESTRUCTURAS

TITULO: UN MODELO MATEMATICO PARA EL ANALISIS DE ESFUERZOS EN MAMPOSTERIAS.

SUMARIO:

Este es un nuevo intento para desarrollar el análisis de esfuerzos en mampostería, considerada como un material no capaz de absorber esfuerzos de tensión, en elementos confinados aisladamente.

El modelo usado es « reticular » lo cual permite obtener la solución por la condición estacionaria de la energía complementaria asociada a un sistema de desigualdades.

El problema que tenemos que resolver es el clásico de programación cuadrática que se vuelve un problema de complementaridad lineal, gracias a la aplicación del teorema de Kuhn y Tucker, de manera que puede ser resuelto por el método simple.

En términos físicos, este último método, permite aplicar algunas fuerzas « restrictivas » en las barras del modelo sujetas a tensión y reanalizar la continuidad por efecto de tales fuerzas, hasta que todos los esfuerzos de tensión se reduzcan a una cantidad despreciable.

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Предмет : СТРУКТУРА

Название : "МАТЕМАТИЧЕСКИЙ ПРИМЕР ДЛЯ ИЗУЧЕНИЯ ДАВЛЕНИЯ ДАННОГО СТРОЕНИЯ."

Краткий конспект :

Мы здесь видим подход к разработке анализа давления в постройках рассматриваемые как "невесомые материалы" благодаря описанию определенных элементов.

Взятый пример является "ретикулярной моделью" позволяющей прийти к заключению, благодаря неподвижному состоянию комплементарной энергии, соединенной с системой неравенства.

Проблема подлежащая к разрешению есть классическая проблема квадратурной программации, которая становится "комплиментарной линейной проблемой" по теореме Куна и Тукера и которую, таким образом, можно разрешить "методом симплицизирования".

Пользуясь терминами взятыми из физики, этот последний метод позволяет применить ограниченные силы к поперечным брусьям модели подвергаемой силам давления и переанализировать непрерывную функцию для получения результата данных сил, до тех пор, пока все силы давления будут сведены до количества бесконечно малой величины.

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TEMA: STRUTTURE

TITOLO: UN MODELLO MATEMATICO PER L'ANALISI DELLO STRESS NELLE COSTRUZIONI IN MURATURA.

SOMMARIO:

In questo saggio viene trattato un approccio allo sviluppo dell'analisi degli sforzi ai quali sono sottoposte le costruzioni in muratura, considerate « materiale non resistente a tensione », caratterizzato da elementi ben definiti.

Il modello usato è un « modello reticolare » che permette di ottenere la soluzione mediante la condizione stazionaria dell'energia complementare, associata ad un sistema di ineguaglianze.

Il problema da risolvere è quello classico della programmazione quadratica, il quale diventa un « problema di complementarietà lineare » che, secondo i teoremi di Kuhn e Tucker, può essere risolto con il « metodo più semplice ».

In termini fisici, quest'ultimo metodo permette di applicare alcune forze « frenanti » ai modelli di spranghe soggetti a tensione, e di rianalizzare l'effetto di tali forze, finché ogni tensione stressante sarà ridotta ad una quantità irrilevante.

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ADHESIVES IN MASONRY STRUCTURES: EXPERIMENTAL ANALYSIS

Introduction

A great deal of interest has been recently devoted to masonry structures, their use and restoration. Such interest seems to be characterized by problems of structure stability and by morphological considerations As far as reliability of structures must be warranted, the particular problems we intend to deal with can show complex and new uncertainties. These are, for instance, in the lacking of complete and detailed codes for the analysis of such structures, for the implementation of research layouts and for the evaluation of available data.

From this point of view, recent and original contributions have been developed about the use of synthetic resins, by means of impregnation " in vacuo ", since such procedures can be very effective in existing masonry works.

In the same time, we may define a process-strategy for the structural restoration of masonry by means of local or extended use of resins, what implies also a new definition of the framework itself. To this aim, several experimental date must be obviously available, as they are necessary to the control of mechanical parameters.

The object of the present study is mainly aimed to a comparison program between the natural material and the processed one.