Chronic defects in masonry vaults: Sabouret's cracks

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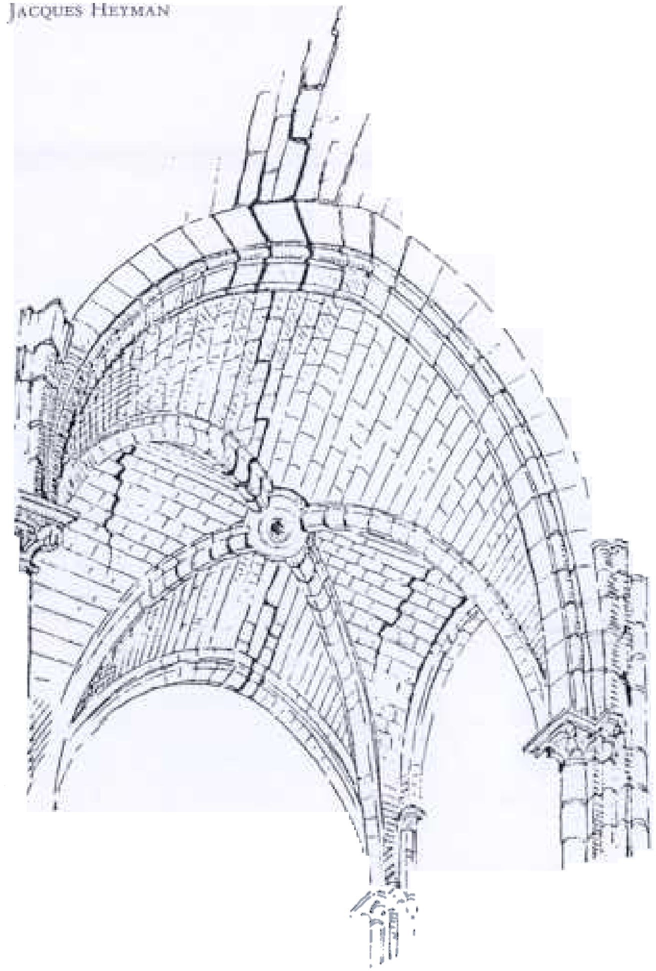

FIG. 1. Typical cracks in Gothic vaults. Pol Abraham distinguished between the tensile cracks near the crown, the 'Sabouret cracks' parallel to the wall ribs, and the separation of the vault from the walls.

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There are three identifiably different types of crack which can occur in masonry vaults. *Fig. 1* reproduces Pol Abraham's drawing[1] of a typical quadripartite bay, and his identification of the cracks, which all run roughly east/west, is as follows.

1. There are the cracks in the main barrel of the vault, in the region of the crown, and often running through the main transverse arches and through the diagonal ribs near the key-stone. For more pointed arches, these cracks occur at some distance away from the crown.
2. There are the cracks that Abraham called *fissures de Sabouret*[2], parallel to the wall ribs but some way in from the wall.
3. There is often complete separation of the vault web from the north and south walls, so that, standing on top of the vault, one may view the floor of the church below.

All three of these types of crack may have existed for many years, or for centuries, in the vaults of a particular church. They do not, in themselves, indicate that the vaults are in a dangerous state; rather, they are all related, and arise from a simple pattern of movement that has occurred in the past and that is not necessarily continuing. The characteristics of masonry that are essential for the formulation of a structural analysis have been noted elsewhere.[3] Briefly, the stresses which arise in masonry in normal construction are extraordinarily low compared with the basic crushing strength of the stone. Indeed the stress levels are so low that it is convenient, and only very slightly unsafe, to assume that the material is, effectively, infinitely strong in compression. On the other hand, the material is very weak in tension; the stones themselves may be strong, but they are assembled with weak mortar, or with no mortar at all. As a balancing assumption about the material behaviour it is convenient, then, but only slightly on the safe side, to regard masonry as incapable of sustaining any tensile stress.

Thus a picture emerges of masonry as an assemblage of small pieces of stone, cut to pack together in a coherent structural form, with that form maintained by compressive forces transmitted within the mass of material, but liable to crack should any tensile forces try to develop. Thus a first requirement of any structural theory of masonry is that it should model accurately this material behaviour. The results, for example, of a conventional theory which assumes similar behaviour in tension and compression, will give results which at best will be only partially relevant to the behaviour of the real structure.

However, there is a deeper requirement for a proper theory of the behaviour of masonry. Masonry structures are obviously 'redundant', in the technical sense that, given a particular set of loads, there can exist many different solutions (in fact an infinite number) for the distribution of force within the structure, any one of which will satisfactorily carry the

[1] Pol Abraham, *Viollet-le-uc et le rationalisme médiéval*, ris, 1934.
[2] Sabouret, V., Les voûtes .rêtes nervurées, *Le génie ʼl*, 3 March 1928.
[3] *See*, for example, Heyman, The stone skeleton, *Int. J. ids Structures*, (1966), ii, 249. the rubber vaults of the ldle Ages, and other mat-, *Gaz. Beaux-Arts*, (1968), 177. *Equilibrium of shell :tures*, Oxford 1977; *The ınry arch*, Chichester 1982.

applied loading. The *actual* pattern of force within a structure will depend on the precise way the structure was built and on the way it is supported externally. In order, then, to calculate this actual state, the engineer needs a great deal of detailed information, some of which by its nature may be difficult, or impossible, to obtain. What the engineer usually does, in fact, is to assume that his structure (as it might be a nave vault) is rigidly attached to rigid supports (the piers and flying buttresses), or, if he is very sophisticated, he may make some notional allowance for flexibility of the supports; he will then assume, almost certainly, that the vault material is elastic and homogeneous; and finally he will obtain his 'solution', either analytically, using perhaps finite elements in association with a computer program, or perhaps experimentally, using a physical model in the laboratory.

Now in the first place this 'solution' is heavily dependent on the engineer's assumptions; very small changes in the way the vault is connected to the piers, or in the flexibility (if any) allowed for the buttressing system, can lead to very large changes in the distribution of forces within the structure. Moreover, even if the engineer did have an exact description of his structure, the mere passage of time would render that description obsolete; a small settlement of a pier, or a 'lurch' of the whole building during a gale, would necessitate a new calculation. It is, in this sense, meaningless to ask for a description of the 'actual' state of the structure, since the state will be different at different times. The best that can be said for a conventional elastic solution is that it does at least describe one possible state of equilibrium for the structure.

In this situation there is comfort to be found in the 'plastic' theorems of structural behaviour. The plastic theory was first developed in response to very similar observations of the behaviour of steel structures. Small constructional defects can, in theory, have very large effects on elastic stress distributions, but it does not seem reasonable to suppose that such small defects can have any real effect on the overall strength of a steel frame. This common-sense view is supported completely by the plastic theorems, of which the essential is the 'safe' or 'lower-bound' or 'anthropomorphic' theorem. Stated simply, it is this: If the engineer can find just one way in which the structure *could* carry its loading, then he may be assured that the structure itself is equally capable of finding such a way. The structure's way may not be the engineer's way, and indeed it will change from time to time, as has been seen, in response to shifts in the external environment, but that a way *will* be found is certain.

Viewed in this light, elastic calculations of stresses are seen not to be meaningful as a way of assessing the stability of a masonry structure. However, masonry is a material which satisfies the fundamental requirements for the application of plastic theory, and the way is open to build on that theory as an alternative to the unnecessary complications of elastic analysis. What is needed is merely the construction of a reasonable

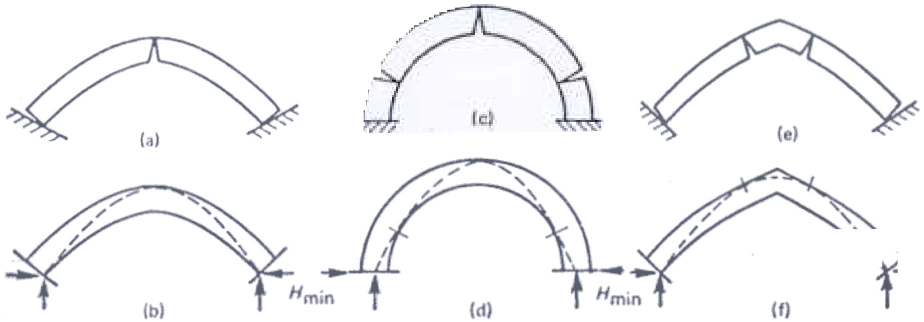


FIG. 2. Crack patterns in masonry arches resulting from increased spans.

distribution of internal forces in equilibrium with the external loads; how this may be done for the masonry vault is seen below.

Fig. 2a shows an incomplete circular arch that was originally fitted perfectly between abutments. Those abutments have spread, and, in order to accommodate itself to the increased span, the arch has cracked as shown, forming 'hinges'. (The drawing is, of course, grossly exaggerated. The cracks could be only hairline, although equally they could, in practice, measure several millimetres.)

The three hinges in *Fig. 2a* have transformed a structure which initially had three redundancies into one which is statically determinate; the three-pin arch is a well-known and perfectly satisfactory structural form. Moreover, whereas the original arch had an infinite number of possible equilibrium states, there is no ambiguity about the condition of the arch in *Fig. 2a*. Clearly the forces transmitted between the pieces of the arch, and maintaining these pieces in equilibrium under the arch's own weight, must act through the hinge points. The line of thrust in the arch, *Fig. 2b*, may in fact be drawn as shown. (The line of thrust represents the shape of an arch that would, in theory, and even if it were of infinitesimal thickness, be able to carry the loading on the actual arch. As long ago as 1675 Robert Hooke pointed out the equivalence, statically, of the problem of the arch and that of the hanging cord. If a flexible string is loaded by a succession of beads representing weights of corresponding successive portions of the arch, then the shape of that string will be the same, upside down, as that of the thrust line of the arch. 'As hangs the flexible line, so but inverted will stand the rigid arch'.)

In *Fig. 3* a full semicircular arch is shown with a possible thrust line contained within its surfaces. The 'actual' position of this thrust line cannot be found without making the assumptions referred to above. The 'safe theorem' states that, so long as a thrust line *can* be drawn lying wholly within the masonry, then the arch will be stable, and no further analysis is necessary. (Clearly, if the arch were thinner—about half the thickness sketched in *Fig. 3*—then it would not be possible to position a thrust line within the masonry. In this case the arch would be of the wrong geometrical shape to carry its own weight, and would collapse on decentering.)

In *Fig. 2c* the arch of *Fig. 3* has suffered an increase of span because of spread of the abutments; *Fig. 2d* shows the limiting position of the thrust line, defining the positions of the cracks in the arch. Again, the arch has become statically determinate by the formation of three hinges.

From *Figs 2e and f* it will be seen that a pointed arch should, theoretically, form four hinges under the same conditions. In fact, it is evident that any slight asymmetry, whether of geometry or of loading, will ensure that only one of the hinges near the crown will form. Two adjacent hinges of this kind may always be thought of, and will usually occur as, a single hinge formed asymmetrically slightly away from the crown.

A simple extension of these basic ideas leads to an understanding of the mechanics of the vault. As a first step towards this understanding, *Fig. 4a* shows the cross-section of a uniform cylindrical barrel vault, drawn roughly to scale (say a vault thickness of 300 mm with a span of 15 m). The vault is supposed to be maintained by external supports, that is by main buttresses, or by flying buttresses transferring the thrust over side aisles. As drawn, the vault is in fact too thin to carry its own weight (in theory, a minimum thickness of about 800 mm is required for a span of 15 m if a semi-circular arch is to be just stable); that is, although the thrust line could be contained within the upper portion of the arch, the thrust would 'escape' from the masonry in regions towards the springings. These regions must therefore be reinforced, and 'fill' is shown in *Fig. 4a* backing the haunches of the barrel, and capable of transferring the thrust to the buttressing system. In actual construction, the fill would be composed of rubble (i.e. unsquared) masonry set in mortar.

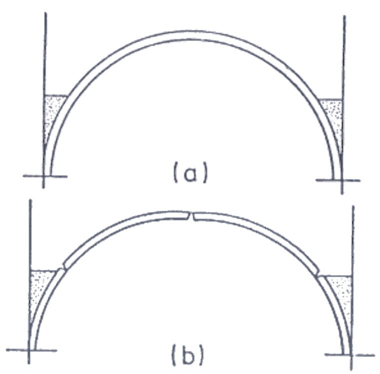
In *Fig. 4b* the buttressing system is supposed to have given way slightly; hinge lines will appear (*cf. Fig. 2c*), although only one of these, that at the crown, will be visible from within the church. Had the barrel vault had a pointed cross-section, then again a single line of cracking would be visible (*cf. Fig. 2e* and the discussion of 'split' hinges), this time slightly away from the crown of the vault. This cracking, at or near the crown, is the first kind of chronic defect referred to at the start of this paper, and represents therefore the trace of a hinge formed as the vault adjusts itself to spread in its supports.

FIG. 3. A possible line of thrust in a circular arch.

The vault of *Fig. 4* is essentially two-dimensional, in that the cross-section was supposed to be the same down the length of the church. *Fig. 5* shows, schematically, a single square bay of a quadripartite vault formed by the intersection of cylindrical barrels. In *Fig. 5a* an elevation of the vault is shown; the fill, which serves the same function as before, is placed in the vaulting conoids (*cf.* the plan of *Fig. 5d.* The vault is of course supposed to extend for several bays, as indicated in the plan). If now the supports of the vault give way, that portion which runs east/west will crack as before, *Fig. 5b*, and the single hinge line at or near the crown, the first kind of chronic defect, will be visible from within the church. The change in overall geometry is accommodated by rotation of the three hinges, with a consequent drop of the crown of the vault.

There is, however, a severe geometrical requirement imposed on the intersecting vault which runs north/south. The horizontal soffit of this vault was built to the original dimension of the span, but the span has now increased. The mismatch in dimensions is zero at the vaulting conoid, and increases to a maximum at the crown, as indicated in *Fig. 5c*; the curve shown would be a sine curve if the original vaults were circular in cross-section, but is of roughly the same shape for vaults of any other practical profile. The general effect of the mismatch is evident in the elevation of *Fig. 5b*; from the left-hand (south) side of the plan of *Fig. 5d*, it will be seen that the whole of the geometrical incompatibility could be taken up by a (sine-shaped) gap opening between the vault and the wall. Such cracking is the third kind of chronic defect in vaults.

However, from the plan and elevation of the south side of the vault it will be seen that the masonry adjacent to the wall is in a state of severe strain. On plan some curvature of the masonry is required, and this could perhaps be accommodated in part by flexing of the vault, depending on the precise proportion of stone and mortar joints. Further, some vertical strain will also be imposed on the north/south vault by the hinge rotations of the east/west vault. It will be appreciated that much of this



IG. 4. Crack pattern in a ylindrical barrel vault esulting from yielding of e buttressing system.

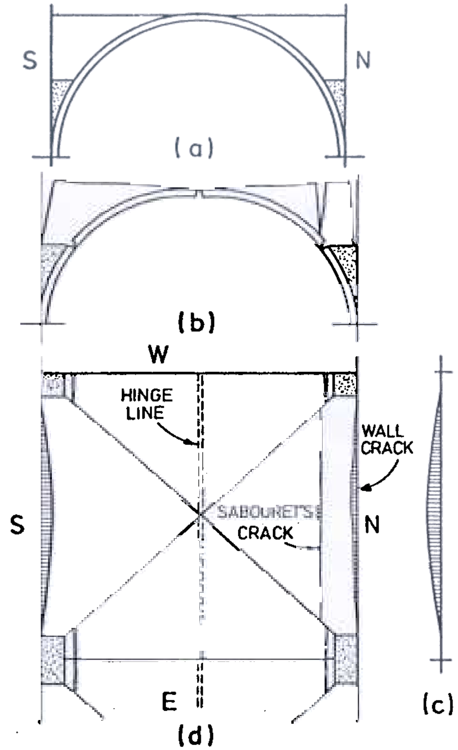


FIG. 5. Crack patterns in one compartment of a quadripartite vault resulting from yielding of the buttressing system.

strain will be relieved if secondary cracks open parallel to the wall cracks, as shown in the right-hand (north) half of the plan in *Fig. 5d.* These are Sabouret's cracks. A Sabouret crack and a wall crack will effectively isolate a portion of the north/south vault, and this portion will then be free to act as a simple arch running east/west, and spanning roughly between adjacent vaulting conoids (although the width in practice of this arch isolated by the cracking will depend also on other factors, including the shape of the main cross-section of the vault).

Thus cracks of the first kind, at or near the crown, are traces of hinge lines in a portion of the vault through which compressive forces are being carried, the forces in fact being approximately perpendicular to the hinge lines. The cracks of the second and third kind, however (Sabouret's cracks and the wall cracks), represent potential or actual complete separation of the masonry; no forces can be transmitted across these fissures, and the forces in fact run parallel to the cracks. The behaviour modelled in *Fig. 5* has been based upon a consideration of an idealized quadripartite cylindrical vault having square bays. The general pathology of all vaults, however, is of exactly the same nature; if they are pointed, then the hinge line at the crown will be displaced, but Sabouret's cracks and wall cracks will still form; nor does a rectangular vaulting bay exhibit a different pattern of behaviour. Further, the arguments apply to groin vaults as well as to rib vaults (and indeed to fan vaults, perhaps supporting a fairly flat spandrel barrel); whether or not the ribs carry a Gothic vault (and this is a celebrated problem in architectural history touched on briefly below), they will be compelled to display the same pattern of hinges as the vault itself when the external geometry changes.

The general crack pattern of *Fig. 1* (or *Fig. 5*) indicates a simple way in which forces may be assessed. In *Fig. 6* a quadripartite bay of vaulting is supported on diagonal ribs. For the purpose of analysis, the vault has been 'sliced', in imagination, into a series of parallel arches, each of a cross-section perhaps like *Fig. 2e*, although much thinner than sketched there, and each springing from the diagonal ribs. If, now, a satisfactory state can be found for this 'sliced' vault, then common sense would indicate (and common sense is in this case supported by the safe theorem of plasticity) that the original unsliced vault would also be satisfactory.

Thus each arch could be analysed in turn, and an estimate made of the loads imposed on the ribs; it turns out that it is very easy to arrive at an understanding of the primary vault forces in this way. However, some order-of-magnitude calculations of a different sort will also serve to give an answer to the question of the function of the ribs.

In the first place, the stresses in a curved arch carrying its own weight are of the order Rw, where R is the (local) radius of the arch and w is the density of the material. It may be noted that this value of stress is independent of the thickness of the arch; if the thickness is doubled, then the weight will be doubled, but so also will be the area of masonry available to resist the forces. A pointed vault over a span of 15 m might have a radius R of about 10 m. If the density of the stone is 2000 kg/m^3, then the product Rw becomes 2 kg/cm^2, to be compared with a crushing strength (for a medium sandstone) of about 400 kg/cm^2. Thus the notion that stresses are extraordinarily low in masonry is supported, at least for vault webs.

However, these remarks apply only to smooth surfaces. At the intersection of the vault webs, that is at the groins (or diagonal ribs), there is a marked increase in stress; a 'crease' in a shell structure is a line of stress concentration. In the sliced model of *Fig. 6* the 'creases' are supposed to be reinforced with diagonal ribs; the sliced arches spring from the backs of these ribs. Now the forces in the ribs could be estimated from a summation of the loads delivered by the sliced arches, but the maximum force in one of the ribs must in any case be of the order of one-quarter of the weight of a vaulting bay. If the bay measures 15 m by 7·5 m, and the vault is 300 mm thick, then the vault might have a total mass of about 120 000 kg, so that each rib might carry about 30 000 kg. A rib measuring

300 mm by 300 mm would thus be working at a stress of about 33 kg/cm^2, still a comfortable level compared with a crushing strength of 400 kg/cm^2.

Now it will be appreciated that a 300 mm by 300 mm rib can be thought of as contained within the main vault webs, themselves of thickness 300 mm. If, then, a vault of these dimensions is built without ribs, the above analysis indicates that the 'background' shell stress in the webs, of about 2 kg/cm^2, will increase sharply in the neighbourhood of the groins to say 30 kg/cm^2. There will be a high stress concentration, but the level is still low compared with the basic strength of the stone, and, provided the intersection of the vault webs is sufficiently regular, and the mortar sufficiently strong, then it is perfectly possible for the vault to succeed, so to speak, in constructing its own skeletal ribs within itself.

The hinging cracks in *Fig. 5d* lead to the mismatch of geometry in the north/south vaults, and these lead in turn to the separation of the vault web along Sabouret's fissures, roughly in line in plan with the hinge cracks. The direction of these fissures indicate that the 'slicing' of *Fig. 6* should give a very reasonable view of the vault behaviour. However, it should be emphasized that the vault remains a highly redundant structure, for which, despite the indications of the cracks, it is still not meaningful to ask for the 'actual' distribution of forces. What can be said is that the pattern of *Fig. 6* is in fact reasonable, and that, much more strongly, whether the pattern is reasonable or not, calculations of vault stability based upon that pattern are safe.

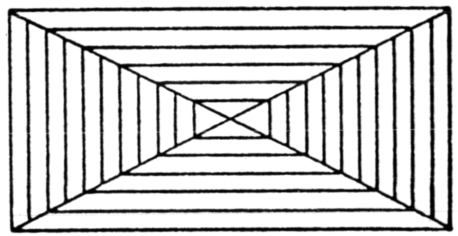


FIG. 6. Vault 'sliced' into parallel arches for purpose of determining primary forces.

Moreover, the general conclusions about the structural function of the ribs will not be affected by different patterns adopted for analysis. A sharp crease in a shell surface, such as occurs in the intersection of curved vault webs, will lead to a sharp stress concentration; if a rib is present at the crease it will help to carry the vault, but it is not absolutely necessary. Thus the diagonal ribs in a quadripartite vault emerge with a clear possible structural function, but they are also, of course, of use during construction. They enable the vault webs to be laid out more easily and allow a good deal of formwork to be dispensed with, and they cover ill-matching intersections of the vault webs where they meet at the groins.

Finally, the rib has sometimes been thought to be aesthetically satisfying, defining visually the 'flow of force' (although not in fact very accurately) in the vault of a Gothic cathedral.

In a quadripartite nave vault formed by intersecting barrels, whether pointed or circular, and having a level soffit, there will be no creases at the nave walls or at the positions of the main transverse arches. Under these circumstances, then, neither the wall arches nor the transverse arches will contribute to the carrying of the vault, and may indeed carry no more than their own weights; the same will be true of ridge ribs if these are present. If the soffit of the vault is not level, as when each vaulting bay is strongly domed, for example, then a crease will appear at the transverse arches, and they will then be called upon to carry some of the load. By contrast, lierne ribs, or any more complex pattern of ribs applied to a smoothly curved vault surface, are purely decorative in function.

Résumé

Il y a trois sortes de fissures qui peuvent se former dans les voûtes de maçonnerie qui, toutes, sont orientées en gros est/ouest. Tout d'abord il peut se former des fissures près du sommet de la voûte: ce sont les 'joints de rupture' visibles seulement par en-dessous. La deuxième et la troisième sorte de fissures aboutissent à la séparation totale de la voûte et de ses nervures; les 'fissures de Sabouret' courent parallèlement aux arcs formerets et il peut y avoir également séparation totale entre l'âme de la voûte et les murs nord et sud (*voir Figs 1 et 5*).

La maçonnerie est un matériau de très forte résistance à la compression mais faible à la tension. La formation de fissures est donc inévitable et une bonne théorie de la structure doit tenir compte de ce fait. De plus, une structure de maçonnerie réagit aux mouvements de ses supports: les piliers peuvent se tasser et les arcs-boutants peuvent avoir un légér jeu. De tels défauts, bien que de peu d'importance en eux-mêmes, peuvent modifier sérieusement l'évaluation de l'élasticité de la structure alors que le bon sens semblerait indiquer que sa solidité n'est pas vraiment mise en question. Une analyse conventionnelle de l'élasticité n'est donc pas suffisante pour comprendre les forces qui entrent en jeu dans une structure de maçonnerie. Le même problème, qui s'était déjà posé pour les structures d'acier, avait abouti à la méthode dite 'plastique' d'analyse; cette méthode se fonde essentiellement sur l'évaluation d'éventuels états d'équilibre de la structure en même temps que—pour les structures de maçonnerie—sur l'examen des possibilités de fissuration. L'arc simple, par exemple, sera sensible à une légère augmentation de sa portée comme le montre la *Fig. 2*.

Si l'on prend l'arc pour modèle, la voûte en tunnel réagira à un léger jeu de ses contreforts par des fissures (*Fig. 4b*). La fissure au sommet (*Fig. 4b ou 2c et 2e*) est le joint de rupture (la première sorte de défaut).

La *Fig. 4* montre un voûtain de voûte quadripartite et les modifications géométriques résultant d'une légère augmentation de portée sont la cause des fissures du deuxième et troisième genre, la 'fissure de Sabouret' et la fissure contre le mur.

Les fissures de la première sorte, les joints de rupture, peuvent transmettre des forces dans l'âme, forces qui agissent à peu près à angle droit des fissures. Mais pour les fissures du deuxième et troisième genre, celles qui divisent complètement la maçonnerie, les forces doivent agir parallèlement aux fissures. On peut donc construire un modèle pour évaluer les forces de tension d'une voûte: la *Fig. 6* montre la coupe d'une voûte en une série d'arcs parallèles qui partent des arcs diagonaux. L'analyse de ce modèle permet d'évaluer la force des nervures qui, bien que plus forte que celle de l'âme de la voûte, est tout de même faible si on la compare à la résistance de la pierre. On peut donc conclure que les nervures d'une voûte peuvent être supprimées sans en causer obligatoirement l'effondrement car leurs forces peuvent être intégrées dans celles des arêtes.

De toutes façons, il n'y a pas d'augmentation ponctuelle de la tension là où la courbe de l'âme de la voûte est continue; en revanche, il y en a aux intersections. Donc, les arcs diagonaux peuvent certainement contribuer à porter le poids d'une voûte de maçonnerie tandis que les arcs formerets et les arcs doubleaux ne portent guère plus que leur propre poids. De même les voussures n'ont-elles qu'un rôle décoratif.

Resumen

En las bóvedas de mampostería pueden ocurrir tres tipos distintos de grieta, todos ellos básicamente en dirección este/oeste. El primer tipo existe cerca del coronamiento de la bóveda y es la grieta 'articulada', visible desde abajo, pero no desde arriba. El segundo y tercer tipo corresponden a la separación total de los lienzos de la bóveda. Las 'grietas Sabouret' corren paralelas a los nervios del muro, y también puede darse la separación completa del lienzo de la bóveda de los muros norte y sur (*Figs. 1 y 5*).

La mampostería resulta muy fuerte en compresión, pero débil en tensión. El agrietamiento es, por lo tanto, inevitable, y una teoría estructural adecuada tiene que fundarse en este comportamiento. Además, una estructura de mampostería se halla sujeta a los movimientos de sus soportes; los pilares pueden experimentar asentamiento y los sistemas de apoyo pueden ceder ligeramente. Estos ligeros defectos pueden afectar grandemente los valores calculados de las fuerzas elásticas, mientras que el sentido común indica que la potencia real de la estructura no se ve afectada. Así pues, debe buscarse algo distinto del análisis elástico convencional para comprender la estructura de la mampostería.

Esta situación ya había surgido en el caso de las estructuras de acero y había hecho nacer el llamado método 'plástico' de análisis, que se basa, esencialmente, en el cálculo de los posibles estados de equilibrio de la estructura, junto con un examen, en al caso de la mampostería, del posible estado de agrietamiento. El arco simple, por ejemplo, responde a una pequeña ampliación de su abertura de la manera indicada en la *Fig. 2*.

Empleando el arco como modelo, la bóveda cilíndrica responde a una pequeña debilidad de su sistema de soporte agrietándose según la Fig. 4b. La grieta del coronamiento de la *Fig. 4b*, o en las *Figs 2c y 2e*, es la grieta 'articulada' (el primer tipo de defecto).

La *Fig. 4* muestra uno de los entrepaños de una bóveda cuatripartita, y los cambios geométricos resultantes del pequeño aumento de abertura conducen al segundo y tercer tipo de defectos, la grieta Sabouret y la de la pared.

Las grietas del primer tipo, las articuladas, pueden transmitir empujes en el lienzo; de hecho, estos empujes actúan básicamente en ángulo recto con las grietas. Para el segundo y tercer tipo de grieta, sin embargo, en los que la mampostería se separa completamente, los empujes deben ir básicamente paralelos a las grietas. De este modo puede construirse un modelo posible para estimar los empujes de las bóvedas; en la *Fig. 6*, la bóveda ha sido 'tajada', de modo imaginario, en una serie de arcos paralelos que nacen de las nerviaciones diagonales. El análisis de este modelo lleva a la evaluación de los empujes en los nervios; aunque los empujes en los nervios resultan altos comparados con los de los lienzos, todavía son bajos comparados con la fuerza básica de la piedra. Así pues, puede llegarse a la conclusión de que pueden quitarse los nervios de una bóveda sin que ésta necesariamente se hunda; los empujes de los 'nervios' pueden acomodarse en la arista de encuentro.

En todo caso, no hay incremento local de empuje en las partes de un lienzo de bóveda que giran con suavidad, sino sólo en las intersecciones de los lienzos. Por lo tanto, las nerviaciones diagonales pueden ciertamente contribuir a sostener una bóveda de mampostería, pero los arcos del muro y los arcos transversales sostienen poco más que su propio peso. De modo parecido, los nervios secundarios tienen una función puramente decorativa.